Again, by general reasoning it should be expected that the DY 4 column will be the same before and after the 10% adjustment. This is clear for AY 1 to AY 4 since the development factor is only change for the 1/0 development. But also, the 10% increase in the DY 0 column is exactly cancelled by the 10% decrease in the average factor for 1/0. The only AY for which the reserve will change is AY 5, since the paid-to-date losses are 6270 instead of 5700, so that the reserve will drop by 570.
A very similar result will occur for case (ii), the mean development factor method.

(b) As a result of the 1.1 increase and decrease in the development factors for 2/1 and 3/2, the DY 4 column is unchanged from the original in both (i) and (ii). The reserve will change as a result of the change in paid-to-date losses. The change will occur for AY 3, DY 2. Instead of the original paid-to date loss of 21,600, it will be 10% higher at 23,760, and the reserve will decrease by 2160.

(c) The B-F reserve for a particular AY is Est. Ult. Loss $\times \left(1 - \frac{1}{f_{ult}}\right)$,

where Est. Ult. Loss $=$ Earned Prem. $\times$ Loss Ratio. Since there are no changes in the Earned Premium and Loss Ratios, the Estimated Ultimate Losses are the unchanged for each AY. Also, the only change in development factors occurs in the 1/0 factor f_1, which changes from 3.22227 based on original data to 2.92933 based on updated data. The f_{ult} for AY 2 , AY 3 and AY 4 are unchanged as they do not involve f_1, so there is no change in the B-F reserve for AY 2, AY 3 or AY 4. The f_{ult} for AY 5 is $f_1 \times f_2 \times f_3 \times f_4$, which is
$3.22227 \times 1.56788 \times 1.33651 \times 1.25 = 8.44025$ based on original data, and is
$2.92933 \times 1.56788 \times 1.33651 \times 1.25 = 7.67295$ based on updated data.

The B-F reserve for AY 5 based on original data is $72,924 \times .67 \times \left(1 - \frac{1}{8.44025}\right) = 43,070,$

and based on updated data it is $72,924 \times .67 \times \left(1 - \frac{1}{7.67295}\right) = 42,491.$ The reserve decreases by 579.

6.   (a) Average development factor table:

Development Factors for Closed Claims

Accident Year	Development Year 1/0	2/1	3/2	4/3	5/4	6/5	7/6
2010	1.500	1.222	1.136	1.080	1.037	1.021	1.014
2011	1.500	1.286	1.185	1.094	1.029	1.011	
2012	1.500	1.250	1.133	1.059	1.028		
2013	1.500	1.185	1.094	1.057			
2014	1.400	1.143	1.063				
2015	1.429	1.133					
2016	1.455						
Average	1.469	1.203	1.122	1.072	1.031	1.016	1.014

Cumulative closed claims table based on average method:

Accident Year	Cumulative Closed Claims Through Development Years 0	1	2	3	4	5	6	7	Estimated Ultimate Number of Claims
2010	600	900	1100	1250	1350	1400	1430	1450	1450
2011	700	1050	1350	1600	1750	1800	1820	1845.45	1845
2012	800	1200	1500	1700	1800	1850	1880.1	1906.42	1906.42
2013	900	1350	1600	1750	1850	1907.59	1938.62	1965.74	1966
2014	1000	1400	1600	1700	1823.13	1879.88	1910.47	1937.19	1937
2015	1050	1500	1700	1907.78	2045.96	2109.65	2143.98	2173.96	2174
2016	1100	1600	1925.15	2160.45	2316.93	2389.06	2427.93	2461.88	2462
2017	1150	1689.37	2032.68	2281.13	2446.35	2522.5	2563.54	2599.39	2599

(b) Cumulative paid loss table:

Accident	Cumulative Paid Losses (1,000's) Development Year							
Year	0	1	2	3	4	5	6	7
2010	150	340	500	650	780	890	960	1010
2011	170	380	570	750	900	1040	1140	
2012	200	530	820	1080	1300	1490		
2013	260	660	970	1220	1420			
2014	350	850	1270	1610				
2015	480	1150	1750					
2016	610	1510						
2017	800							

Payment per claim table based on cumulative closed claims and payments:

Accident	Payment Per Ultimate Claim Through Development Year Development Year							
Year	0	1	2	3	4	5	6	7
2010	103.448	234.483	344.828	448.276	537.931	613.793	662.069	696.552
2011	92.1409	205.962	308.943	406.504	487.805	563.686	617.886	
2012	104.932	278.069	430.22	566.632	682.057	781.742		
2013	132.248	335.707	493.388	620.549	722.279			
2014	180.692	438.823	655.653	831.182				
2015	220.791	528.979	804.968					
2016	247.766	613.323						
2017	307.811							

Entries in this table are not in 1,000's. For instance the AY 2010, DY 0 entry is $\frac{150{,}000}{1450} = 103.448$. Note that this is the "average payment" in AY 201, DY per ultimate number of claims.

The average development factors for the previous table are:

	Payment Per Ultimate Claim Development Factors						
Accident	Development Year						
Year	1/0	2/1	3/2	4/3	5/4	6/5	7/6
2010	2.267	1.471	1.300	1.200	1.141	1.079	1.052
2011	2.235	1.500	1.316	1.200	1.156	1.096	
2012	2.650	1.547	1.317	1.204	1.146		
2013	2.538	1.470	1.258	1.164			
2014	2.429	1.494	1.268				
2015	2.396	1.522					
2016	2.475						
Average	2.427	1.501	1.292	1.192	1.148	1.087	1.052

Estimated (boldface below diagonal) payment per ultimate claims:

	Estimated Payment Per Ultimate Claim Through Development Year							
Accident	Development Year							
Year	0	1	2	3	4	5	6	7
2011	92.1409	205.962	308.943	406.504	487.805	563.686	617.886	**650.016**
2012	104.932	278.069	430.22	566.632	682.057	781.742	**849.753**	**893.941**
2013	132.248	335.707	493.388	620.549	722.279	**829.176**	**901.314**	**948.183**
2014	180.692	438.823	655.653	831.182	**990.769**	**1137.4**	**1236.36**	**1300.65**
2015	220.791	528.979	804.968	**1040.02**	**1239.7**	**1423.18**	**1546.99**	**1627.44**
2016	247.766	613.323	**920.597**	**1189.41**	**1417.78**	**1627.61**	**1769.21**	**1861.21**
2017	307.811	**747.111**	**1121.41**	**1448.87**	**1727.05**	**1982.65**	**2155.14**	**2267.21**

Reserve calculation results in a reserve of 11,543,450:

Accident Year	Paid-to date per ultimate	Ultimate loss per claim	Liability per claim	Ultimate Number of Claims	Reserve for Year 1000's
2010	696.552	696.552	0	1450	0
2011	617.886	650.016	32.1301	1845.45	59.2946
2012	781.742	893.941	112.199	1906.42	213.898
2013	722.375	948.183	225.808	1965.74	443.879
2014	831.182	1300.65	469.465	1937.19	909.442
2015	804.968	1627.44	822.47	2173.96	1788.02
2016	613.323	1861.21	1247.89	2461.88	3072.16
2017	307.811	2267.21	1959.4	2599.39	5093.25
Total Reserve					11579.9

(c) Development factors for cumulative paid losses:

	Development Factors for Cumulative Paid Losses						
Accident Year	Development Year						
	1/0	2/1	3/2	4/3	5/4	6/5	7/6
2010	2.267	1.471	1.300	1.200	1.141	1.079	1.052
2011	2.235	1.500	1.316	1.200	1.156	1.096	
2012	2.650	1.547	1.317	1.204	1.146		
2013	2.538	1.470	1.258	1.164			
2014	2.429	1.494	1.268				
2015	2.396	1.522					
2016	2.475						
Average	2.427	1.501	1.292	1.192	1.148	1.087	1.052

Estimated cumulative paid losses and reserves:

Estimated Cumulative Paid Losses (1,000's)									
Accident	Development Year								Reserve
Year	0	1	2	3	4	5	6	7	
2010	150	340	500	650	780	890	960	1010	0
2011	170	380	570	750	900	1040	1140	1199.28	59.28
2012	200	530	820	1080	1300	1490	1619.63	1703.85	213.851
2013	260	660	970	1220	1420	1630.16	1771.98	1864.13	444.127
2014	350	850	1270	1610	1919.12	2203.15	2394.82	2519.35	909.355
2015	480	1150	1750	2261	2695.11	3093.99	3363.17	3538.05	1788.05
2016	610	1510	2266.51	2928.33	3490.57	4007.17	4355.8	4582.3	3072.3
2017	800	1941.6	2914.34	3765.33	4488.27	5152.54	5600.81	5892.05	5092.05
			Total Reserve						11579

The reserves are 11,579,000, which is the same as the reserves found in part (b). It is no coincidence that this is the case. What has happened is that the entries across the row for a particular year in the payment per claim table in (b) is equal to the corresponding entry in the row for that year in the cumulative paid losses table in (c) divided by the estimated ultimate number of claims for that year found in part (a). For instance for 2014, if we divide each of the first three entries by 1937.19, which is the estimated ultimate number of claims for 2014 (and multiply by 1000), we get

$$1000 \times \frac{350}{1937.19} = 180.67 \,, \; 1000 \times \frac{8500}{1937.17} = 438.78 \,, \; 1000 \times \frac{1270}{1937.17} = 655.59 \,.$$

It follows that the development factor table will be identical for each of those tables, and the estimates produced by those development factors will be identical. The method in (b) is the same as the Chain Ladder method and the information on the number of claims is irrelevant. This problem was fashioned after Exercise 3.10 in the text, but this equivalence was not mentioned in the text solution to that problem in the text. Another point to note is that the method in text Exercise 3.10 is not the same as the example given at the start of Section 3.6.5 in the text. In that example, a table is calculated of cumulative loss payments divided by cumulative closed claim numbers for each AY, DY (Table 3.10 in the text).

(d) The cumulative paid loss table is found at the start of the solution to part (b) above. The cumulative closed claims table is the non-bold part of the second table in the solution to part (a) above. We divide entry by entry to get

Accident	Average Cumulative Paid Losses Development Year							
Year	0	1	2	3	4	5	6	7
2010	250	377.778	454.545	520	577.778	635.714	671.329	696.552
2011	242.857	361.905	422.222	468.75	514.286	577.778	626.374	
2012	250	441.667	546.667	635.294	722.222	805.405		
2013	288.889	488.889	606.25	697.143	767.568			
2014	350	607.143	793.75	947.059				
2015	457.143	766.667	1029.41					
2016	554.545	943.75						
2017	695.652							

The development factor table from this table with average development factor at the bottom is

Accident	Average Cumulative Paid Losses Development Factors Development Year						
Year	1/0	2/1	3/2	4/3	5/4	6/5	7/6
2010	1.511	1.203	1.144	1.111	1.100	1.056	1.038
2011	1.490	1.167	1.110	1.097	1.123	1.084	
2012	1.767	1.238	1.162	1.137	1.115		
2013	1.692	1.240	1.150	1.101			
2014	1.735	1.307	1.193				
2015	1.677	1.343					
2016	1.702						
Average	1.653	1.250	1.152	1.112	1.113	1.070	1.038

Applying the average development factors gives the estimate average cumulative paid losses:

Estimated Average Cumulative Paid Losses							
Accident	Development Year						
Year	0	1	2	3	4	5	6	7
2010	250	377.778	454.545	520	577.778	635.714	671.329	696.552
2011	242.857	361.905	422.222	468.75	514.286	577.778	626.374	**650.176**
2012	250	441.667	546.667	635.294	722.222	805.405	**861.783**	**894.531**
2013	288.889	488.889	606.25	697.143	767.568	**854.303**	914.104	948.84
2014	350	607.143	793.75	947.059	**1053.13**	**1172.13**	**1254.18**	1301.84
2015	457.143	766.667	1029.41	**1185.88**	1318.7	**1467.71**	**1570.45**	1630.13
2016	554.545	943.75	**1179.69**	1359	1511.21	1681.97	1799.71	1868.1
2017	695.652	**1150.2**	**1437.75**	**1656.29**	1841.79	2049.92	2193.41	2276.76

The estimated ultimate loss for a particular AY is the DY 7 entry for that AY multiplied by the DY 7 entry from the cumulative closed claims table. For instance for AY 2014, the estimated ultimate loss is $1301.84 \times 1937 = 2,521,664$ (entered as 2521.67 (due to roundoff error in the excel file) in 1000's in the following table). The reserve for that year is, as usual, the estimated ultimate loss minus the losses paid to date (found in the cumulative paid loss table). The resulting reserve is 11,632,100.

	Est. Avg. Ult		Est. Avg		Est. Ult.	Paid to	Reserv
Accident	Cumulative		Ult. Closed		Loss	Date	
Year	Loss		Claims		1000's	1000's	
2010	696.552		1450		1010		
2011	**650.176**		1845		1199.57	1140	59.5744
2012	**894.531**		1906		1704.98	1490	214.976
2013	**948.84**		1966		1865.42	1420	445.419
2014	**1301.84**		1937		2521.67	1610	911.667
2015	**1630.13**		2174		3543.91	1750	1793.91
2016	**1868.1**		2462		4599.27	1510	3089.27
2017	**2276.76**		2599		5917.3	800	5117.3
							11632.1

7.   Step 1. We find the incremental loss severity payments. The is the table created by dividing the incremental paid loss table by the incremental closed claim table given at the start of Problem 5.

Incremental Severity Through Development Years (not 1000's)

AY\DY	0	1	2	3	4	5	6	7
2010	250	633.333	800	1000	1300	2200	2333.33	2500
2011	242.857	600	633.333	720	1000	2800	5000	
2012	250	825	966.667	1300	2200	3800		
2013	288.889	888.889	1240	1666.67	2000			
2014	350	1250	2100	3400				
2015	457.143	1488.89	3000					
2016	554.545	1800						
2017	695.652							

Step 2. Adjust incremental severity to 2017 level using 4% trend rate and calculate the average for each DY. The resulting table is:

Trended Incremental Severity Through Development Years (not 1000's)

AY\DY	0	1	2	3	4	5	6	7
2010	328.983	833.423	1052.75	1315.93	1710.71	2895.05	3070.51	3289.83
2011	307.292	759.191	801.369	911.03	1265.32	3542.89	6326.6	
2012	304.163	1003.74	1176.1	1581.65	2676.64	4623.28		
2013	337.959	1039.87	1450.62	1949.76	2339.72			
2014	393.702	1406.08	2362.21	3824.54				
2015	494.446	1610.38	3244.8					
2016	576.727	1872						
2017	695.652							
Average	429.866	1217.81	1681.31	1916.58	1998.1	3687.07	4698.55	3289.83

Step 3. For DY 1 to DY 7 trend backward from the average to fill the lower triangle in the original incremental severity table.

Detrended Incremental Severity Through Development Years (not 1000's)

AY\DY	0	1	2	3	4	5	6	7
2010	250	633.333	800	1000	1300	2200	2333.33	2500
2011	242.857	600	633.333	720	1000	2800	5000	**2600**
2012	250	825	966.667	1300	2200	3800	**3861.87**	**2704**
2013	288.889	888.889	1240	1666.67	2000	**3151.73**	**4016.34**	**2812.16**
2014	350	1250	2100	3400	**1776.3**	**3277.8**	**4176.99**	**2924.65**
2015	457.143	1488.89	3000	**1771.99**	**1847.35**	**3408.91**	**4344.07**	**3041.63**
2016	554.545	1800	**1616.64**	**1842.87**	**1921.25**	**3545.26**	**4517.84**	**3163.3**
2017	695.652	**1217.81**	**1681.31**	**1916.58**	**1998.1**	**3687.07**	**4698.55**	**3289.83**

Step 4. Calculate the percentage of claims closed through development year table from the cumulative closed claims table. This is done in the same way Table 41.11 was calculated from Table 41.5 in the Section 41. The table is:

	Percentage of Claims Closed Through Development Years							
AY\DY	0	1	2	3	4	5	6	7
2010	41.38%	35.29%	36.36%	42.86%	50%	50%	60%	100%
2011	37.94%	30.57%	37.74%	50.51%	61.22%	53%	44%	
2012	41.96%	36.15%	42.47%	49.21%	48.54%	47.17%		
2013	45.78%	42.21%	40.58%	41%	46.30%			
2014	51.63%	42.69%	37%	30%				
2015	48.30%	40%	29.67%					
2016	44.68%	36.71%						
2017	44.25%							
Average	44.49%	37.66%	37.30%	42.72%	51.52%	50.05%	52.00%	100%

Step 5. We now estimate the incremental closed claims for each AY and DY in the region below the diagonal applying the average percentages in the table in Step 4 to fill in the entries (bold) below the diagonal in the Incremental Closed Claims table, just as Table 41.12 was found in Section 41.

	Estimated Incremental Closed Claims Through Development Years								
Accident	Development Year								
Year	0	1	2	3	4	5	6	7	
2010	600	300	200	150	100	50	30	20	1450
2011	700	350	300	250	150	50	20	**25**	1845
2012	800	400	300	200	100	50	**29**	**27**	1906
2013	900	450	250	150	100	**58**	**30**	**28**	1966
2014	1000	400	200	100	**122**	**57**	**30**	**28**	1937
2015	1050	450	200	**202**	**140**	**66**	**34**	**32**	2174
2016	1100	500	**322**	**230**	**159**	**75**	**39**	**36**	2462
2017	1150	**546**	**337**	**241**	**167**	**78**	**41**	**38**	2599

Step 6. Calculate projected incremental loss payments. This is done by multiplying bold entries in the table in Step 5 by the corresponding detrended incremental severities, the bold entries in the table in Step 3.

AY\DY		0	1	2	3	4	5	6	7	Reserve
		Projected Incremental Loss Payments (1,000's)								
2011	2011								65	65
2012	2012							111.994	73.008	185.002
2013	2013						182.8	120.49	78.7405	382.031
2014	2014				216.709	186.834	125.31	81.8901	610.743
2015	2015			357.942	258.629	224.988	147.699	97.3322	1086.59
2016	2016		520.559	423.86	305.478	265.895	176.196	113.879	1805.87
2017	2017	664.926	566.601	461.896	333.682	287.592	192.641	125.014	2632.35
	Total Reserve									6767.58

The total across each AY row is the reserve for that year, and the sum of those is the total reserve of 6,603,120.